Fluids of light with *driven-dissipative* vs. *unitary* quantum dynamics

thermalization, quantum quenches, evaporation & co.

And an excursion into synthetic dimensions

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Experimental collaboration with:

- D. Vocke, D. Faccio (Heriot-Watt, Edinburgh)
- A. Bramati, Q. Glorieux, E. Giacobino (LKB, Paris)
- L. Pavesi (Univ. Trento)
Why not hydrodynamics of light?

Light field/beam composed by a huge number of photons
• in vacuo photons travel along straight line at c
• (practically) do not interact with each other
• in cavity, collisional thermalization slower than with walls and losses

=> optics typically dominated by single-particle physics

In photonic structure:
$\chi^{(3)}$ nonlinearity → photon-photon interactions
Spatial confinement → effective photon mass

=> collective behaviour of a quantum fluid

Many experiments so far:
BEC, superfluidity, quantum magnetism...

In this talk: a few selected topics
- Quantum fluids of light with unitary dynamics
- (if time permits) Synthetic dimensions for photons
Standing on the shoulders of giants

Laserlight—First Example of a Second-Order Phase Transition Far Away from Thermal Equilibrium*

R. Graham and H. Haken
I. Institut für theoretische Physik der Universität Stuttgart

Received April 23, 1970

We solve the functional Fokker-Planck equation established in a previous paper in the vicinity of laser threshold. The stationary solution is obtained explicitly in the form \( P = N \exp [-\varphi(U, U^*)] \). \( \varphi \) has exactly the same form as the Ginzburg-Landau expression for the free energy of a superconductor, if the pair wave function is replaced by the electromagnetic field amplitude \( U \). This gives us the key for a thermodynamic reinterpretation of all laser phenomena.

In particular the laser threshold appears as a second-order phase transition in all details. It is indicated that our theory provides a new formalism also for the Ginzburg-Landau theory.

- Vortices and Defect Statistics in Two-Dimensional Optical Chaos

F. T. Arecchi, (a) G. Giacomelli, P. L. Ramazza, and S. Residori
Istituto Nazionale di Ottica, Largo E. Fermi, 6, 50125 Firenze, Italy
(Received 1 April 1991)

We present the first direct experimental evidence of topological defects in nonlinear optics. For increasing Fresnel numbers \( F \), the two-dimensional field is characterized by an increasing number of topological defects, from a single vortex, up to a large number of vortices with zero net topological charge. At variance with linear scattering from a fixed phase plate, here the defect pattern evolves in time according to the nonlinear dynamics. We assign the scaling exponents for the mean number of defects, their mean separation, and the charge unbalance as functions of \( F \), as well as the correlation time of the defect pattern.


Optique/Optics

Diffraction non linéaire

Yves Pomeau et Sergio Rica

Résumé—Une expérience classique en mécanique des fluides est la formation de structures vorticales à l’arrière d’un obstacle, comme par exemple l’écoulement de Bénard-von Kármán. Est-il possible d’imaginer une expérience similaire en optique ? C’est-à-dire, en illuminant un obstacle pourrait-on engendrer des structures tourbillonnaires caractéristiques d’un régime pré-turbulent ? Cette Note est consacrée au problème de la génération de vorticité dans les ondes électromagnétiques.

Hydrodynamic phenomena in laser physics: Modes with flow and vortices behind an obstacle in an optical channel

M. Vaupeel, K. Stalinaus, and C. O. Weiss
Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany
(Received 16 February 1995; revised manuscript received 20 February 1996)

The transverse patterns of an active resonator with cylindrical optics are investigated. This resonator configuration corresponds to a ‘channel’ form of the potential for the ‘photon fluid’. Simultaneous emission of different transverse modes along the channel, periodic nucleation of vortices in the form of a vortex street (vortices of alternating senses of rotation appearing in a flow behind an obstacle), accelerated flow in a ‘tilted channel’ and destabilization of the one-directional flow in the channel are demonstrated and interpreted in terms of tilted waves and beating of channel modes, as well as in fluid terms, illustrating the fluid dynamics correspondence of class-A lasers. [S1650-2947/96/02407-9]

And of course many others:
Coulet, Gil, Rocca, Brambilla, Lugiato...
**Dissipative vs. conservative quantum fluids of light**

**Planar microcavities & cavity arrays**

Pump needed to compensate losses: driven-dissipative dynamics in real time stationary state ≠ thermodyn. Equilibrium

Driven-dissipative CGLE evolution

\[ i \frac{dE}{dt} = \left\{ \omega_o - \frac{\hbar \nabla^2}{2m} + V_{ext} + g|E|^2 + i \frac{P_0}{2(1+\alpha|E|^2)} - \gamma \right\} E + F_{ext} \]

State of in-cavity field copied to emission Quantum correl. sensitive to dissipation

**Propagating geometry**

Monochromatic beam: Incident beam sets initial condition @ z=0

Conservative paraxial propagation → Gross-Pitaevskii eq @ mean-field

\[ i \frac{dE}{dz} = \left\{ -\frac{\hbar \nabla_{xy}^2}{2\beta} + V_{ext} + g|E|^2 \right\} E \]

- \( V_{ext} \), g proportional to -(\(\epsilon(r)\))-1 and \( \chi^{(3)} \)
- Mass → diffraction (xy)

But... what happens to quantum dynamics?
**Dissipative QFL's: already many success**

**Polariton BEC**

**Polariton superfluidity**


**Topologically protected edge states**
First expts with (almost) conservative QFL’s

Dispersive superfluid-like shock waves


Chiral edge states in (photonic) Floquet topological insulator

Many more expts in Alexander Szameit's lectures

Bogoliubov dispersion of collective excitations

D. Vocke et al., Optica (2015)

Quantum simul. of 2-body physics
Mukherjee et al., arXiv:1604.00689

Hydrodynamic nucleation of quantized vortices

D. Vocke et al., arXiv:1511.06634
Frictionless flow of superfluid light (I)

All superfluid light experiments so far:

- Planar microcavity device with stationary obstacle in flowing light
- Measure response on the fluid density/momentum pattern
- Obstacle typically is defect embedded in semiconductor material
- Impossible to measure mechanical friction force exerted onto obstacle

Propagating geometry more flexible:

- Obstacle can be solid dielectric slab with different refractive index
- Immersed in liquid nonlinear medium, so can move and deform
- Mechanical force measurable from magnitude of slab deformation

Frictionless flow of superfluid light (II)

Numerics for propagation GPE of monochromatic laser:

\[ i \partial_z E = -\frac{1}{2\beta} \left( \partial_{xx} + \partial_{yy} \right) E + V(r) E + g |E|^2 E \]

with \( V(r) = -\beta \Delta \varepsilon(r)/(2\varepsilon) \) with rectangular cross section and \( g = -\beta \chi^{(3)}/(2\varepsilon) \)

For growing light power, superfluidity visible:

- Intensity modulation disappears
- Suppression of opto-mechanical force

Fused silica slab as obstacle

→ deformation almost in the \( \mu \text{m} \) range

Experiment in progress

→ surrounding medium in fluid state
  but local nonlinearity (e.g. atomic gas)

Condensation of classical waves

Monochromatic beam but spatially noisy profile

Slow nonlinearity → remains monochromatic
Evolution during propagation → classical GPE

Thermalizes to condensate plus thermal cloud with Rayleigh-Jeans $1/k^2$ high-$k$ tail

- What about quantum effects?
- How to recover Planckian?

Sun et al., Nature Physics 8, 470 (2012)
How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field

\[ i \frac{\partial E}{\partial z} = - \frac{1}{2\beta_0} \left( \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \right) - \frac{1}{2D_0} \frac{\partial^2 E}{\partial t^2} + V(r)E + g|E|^2E \]

Propagation coordinate \( z \rightarrow \) time

Physical time \( \rightarrow \) extra spatial variable, dispersion \( D_0 \rightarrow \) temporal mass

Upon quantization \( \rightarrow \) conservative many-body evolution in \( z \):

\[ i \frac{d}{dz} |\psi\> = H |\psi\> \]

with

\[ H = N \iiint dx \, dy \, dt \left[ \frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + V \hat{E}^\dagger \hat{E} + \hat{E}^\dagger \hat{E}^\dagger \hat{E} \hat{E} \right] \]

Same \( z \) commutator

\[ [\hat{E}(x,y,t,z), \hat{E}^\dagger(x',y',t',z)] = \frac{c \hbar \omega_0 v_0}{\epsilon} \delta(x-x') \delta(y-y') \delta(t-t') \]


See also old work by Lai and Haus, PRA 1989
Dynamical Casimir emission at quantum quench (I)

Monochromatic wave @ normal incidence
Slab of weakly nonlinear medium
→ Weakly interacting Bose gas at rest

Air / nonlinear medium interface
→ sudden jump in interaction constant when moving along $z$

Mismatch of Bogoliubov ground state in air and in nonlinear medium
→ emission of phonon pairs at opposite $k$ on top of fluid of light

Propagation along $z$
→ conservative quantum dynamics

Important question: what is quantum evolution at late times? Thermalization?

P.-E. Larré and IC, PRA 92, 043802 (2015)
Dynamical Casimir emission at quantum quench (II)

Observables:

- Far-field → correlated pairs of photons at opposite angles
- Near-field → peculiar pattern of intensity noise correl.

First peak propagates at the speed of sound $c_s$

May simulate dynamical Casimir effect & fluctuations in early universe

Pump & probe expt for speed of sound $c_s$:

- $c_s^{xy}$ (Heriot-Watt – Vocke et al. Optica '15)
- $c_s^t$ (Trento, in progress)

Quantum dynamics most interesting in strongly nonlinear media, e.g. Rydberg polaritons

P.-E. Larré and IC, PRA 92, 043802 (2015)
A potentially important technological issue...

Long-distance fiber-optic set-ups → telecom over distances ~$10^4$ km

Can optical coherence be preserved?

Several disturbing effects:

• (extrinsic) fluctuations of fiber temperature, length, etc.

• (intrinsic) Fiber material has some (typically weak) $\chi^{(3)}$
  Shot noise on photon number gives fluctuations of $n_{\text{refr}} \sim n_0 + \chi^{(3)} I$

Statistical mechanics suggests that phase fluctuations destroy 1D BEC

→ light at the end of fiber has lost its (temporal) phase coherence

Is this intuitive picture correct? How to tame phase decoherence?
“Pre-thermalized” 1D photon gas

Perfectly coherent light injected into 1D optical fiber:
- quantum quench of interactions $\sim \chi^{(3)}$
- pairs of Bogoliubov excitations generated

Resulting phase decoherence in $g^{(1)}(t-t')$:
- Exponential decay at short $|t-t'| < 2z / c_s$
  ($c_s =$ speed of Bogol. sound)
- Plateau at long $|t-t'| > 2z / c_s$
- Low-k modes eventually tends to thermal $T_{\text{eff}} = \mu / 2$
- Hohenberg-Mermin-Wagner theorem prevents long-range order in 1D quasi-condensates at finite $T$

Effect small for typical Si fibers, still potentially harmful on long distances
Decoherence slower if tapering used to “adiabatically” inject light into fiber

Related cold atom expts by J. Schmiedmayer
when 1D quasi-BEC suddenly split in two

**A quite generic quantum simulator**

Quantum many-body evolution in $z$:

$$i \frac{d}{dz} |\psi> = H |\psi> \quad \text{with:} \quad H = N \iiint dx \, dy \, dt \left[ \frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + V \hat{E}^\dagger \hat{E} + \hat{E}^\dagger \hat{E} \hat{E} \hat{E} \right]$$

- Physical time $t$ plays role of extra spatial coordinate
- Same $z$ commutator: $[\hat{E}(x, y, t, z), \hat{E}^\dagger(x', y', t', z)] = \frac{c}{\epsilon} \frac{\hbar \omega_0 V_0}{\epsilon} \delta(x-x') \delta(y-y') \delta(t-t')$

Clever design of $V(x, y, z) \rightarrow$ simulate wide variety of physical systems:

- Arbitrary splitting/recombination of waveguides $\rightarrow$ quench of tunneling
- Modulation along $z \rightarrow$ Floquet topological insulators
- In addition to photonic circuit $\rightarrow$ many-body due to photon-photon interactions
- On top of moving fluid of light $\rightarrow$ simulate general relativistic QFT

P.-E. Larré, IC, PRA 92, 043802 (2015)
Evaporative cooling of light

Quantum Hamiltonian under space-z / time-t mapping:

\[
H = N \iiint dx \, dy \, dt \left[ \frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + g \hat{E}^\dagger \hat{E}^\dagger \hat{E} \hat{E} \right]
\]

In 3D bulk crystal after long propagation distances:

- equilibration in transverse k and frequency \( \omega \) leads to Bose-Einstein distribution (in contrast to Rayleigh-Jeans of expts. so far; no UV pathologies)
- temperature and chemical potential fixed by incident distribution \( I(k, \omega) \)

Harmonic trap in xy plane + selective absorption of most energetic particles:

- Energy redistributed by collisions; photon gas evaporatively cooled
- Incident incoherent (in both space and time) field eventually gets to BEC state
- NOTE: fast and coherent optical nonlinearity \( \chi^{(3)} \) essential !!

Novel source of coherent light

Part II: Towards higher dimensions in driven-dissipative coupled microcavity systems
What about higher dimensions?

Generalize of semiclassical equations to 4D:

\[
\begin{align*}
\dot{r}^\mu(k) &= \frac{\partial\mathcal{E}(k)}{\partial k_\mu} - \dot{k}_\nu \Omega^{\mu\nu}(k) \\
\dot{k}_\mu &= -E_\mu - \dot{r}^\nu B_{\mu\nu},
\end{align*}
\]

Integrate current over filled bands:

- 2D quantized Hall current depends on 1st Chern number

\[
j^y = \frac{E_x}{(2\pi)^2} \int_T \Omega d^2k = \frac{\nu_1}{2\pi} E_x \quad \text{analogous to} \quad j^y = \nu \frac{e^2}{h} \quad \text{well known in IQHE}
\]

- 4D magneto-electric response depends on 2nd Chern number (non-zero in d≥4)

\[
j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{T^4} \Omega^{\mu\nu} d^4k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}
\]

\[
\nu_2 = \frac{1}{4\pi^2} \int_{T^4} \Omega^{xy} \Omega^{zw} + \Omega^{wx} \Omega^{yz} + \Omega^{zx} \Omega^{yw} d^4k
\]

How to create 4D system with atoms?

Internal state → Synthetic dimension w

Raman processes give tunneling along w
Spatial phase of Raman beams give Peierls phase in $xw, yw, zw$
Standard synthetic-B in $xy$ and/or $yz$ and/or $zx$

First experimental realization:
• 1+1 dimens. using 3 spin states
• Cyclotron + Reflection on edges

Stuhl et al., arXiv:1502.02496
**Numerical validation of 4D QH effect**

Numerical simulation of full wave equation
Add weak E and B fields
Results in good agreement with semiclassics
Allowed E,B are enough to see the effect

\[
j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{T^4} \Omega^{\mu\nu} d^4 k + \frac{\nu_2}{4\pi^2} \epsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}
\]

\[
\nu_2 = \frac{1}{4\pi^2} \int_{T^4} \Omega^{xy} \Omega^{zw} + \Omega^{wx} \Omega^{yz} + \Omega^{zx} \Omega^{yw} d^4 k
\]

How to create synthetic dimensions for photons?

Different modes of ring resonators → synthetic dimension $w$

Tunneling along synthetic $w$:
- strong beam modulates resonator $\varepsilon_{ij}$ at $\omega_{\text{FSR}}$ via optical $\chi^{(3)}$
- neighboring modes get linearly coupled
- phase of modulation → Peierls phase along synthetic $w$

Peierls phase along $x,y,z$ → Hafezi's ancilla resonators

Extends Fan's idea of synthetic gauge field via time-dependent modulation (Nat. Phys. 2008)

1+1 array: chiral edge states & optical isolation

1 (physical) + 1 (synthetic) dimensions: Hofstadter model
- Bulk topological invariant → Chern number
  - measured via Integer Quantum Hall effect
  - driven-dissipative steady-state:
    “photon current” → displaced intensity distrib.

- Chiral states on edges:
  - Physical edges along x
  - Synthetic edges via design of $\varepsilon(\omega)$
    (e.g. inserting absorbing impurities in chosen sites)
    → topologically protected optical isolator


3+1 array: 4D Quantum Hall physics

4D magneto-electric response
Nonlinear integer QH effect

Driven-dissipative steady state

Lateral shift in response to external synth-E and synth-B:
- only present with both E & B
- proportional to 2nd Chern

Subtleties in relating shift to QH current

Price et al., On the measurement of Chern numbers through center-of-mass responses, arXiv:1602.01696
to appear on PRB as Editor’s suggestion

\[ j^\mu = E^\nu \frac{1}{(2\pi)^4} \int_{T^4} \Omega^{\mu\nu} \, d^4 k + \frac{\nu_2}{4\pi^2} \epsilon^{\mu\alpha\beta\nu} E^\nu B_{\alpha\beta} \]
\[ \nu_2 = \frac{1}{4\pi^2} \int_{T^4} \Omega^{xy} \Omega^{zw} + \Omega^{wx} \Omega^{yz} + \Omega^{zx} \Omega^{yw} \, d^4 k \]

Conclusions and perspectives

Dilute photon gas
GP-like equation

2000-6 → BEC in exciton-polaritons gas in semiconductor microcav.
2008-10 → superfluid hydrodynamics effects observed
2009-13 → synthetic gauge field for photons and topologically protected edge states observed
2014- → revival of paraxial nonlinear optics in the new perspective of propagating fluids of light

- Optical microcavity systems are unavoidably lossy → driven-dissipative, non-equilibrium dynamics extra quantum noise due to dissipation but also interesting new possibilities
- Bulk cavityless geometries: paraxial propagation → conservative dynamics
time plays role of 3rd dimension: quantum quenches, quantum thermalization, evaporative cooling experimentally challenge → strong nonlinearities. Most promising: Rydberg-EIT in atomic gases

Many other on-going directions:
- strongly correlated photon gases: 1D Tonks-Girardeau gas; 2D Laughlin states with synthetic B
- Momentum-space magnetism → naturally toroidal geometry, possibility of inserting flux
- high-D photonics: new topological effects & device applic. More protected quantum info!

Price et al, PRL 113, 190403 (2014); Berceau et al, PRA 93, 013827 (2016); Ozawa et al, PRA 92, 023609 (2015); PRB 93, 195201 (2016)
If you wish to know more...


Or, even better, come and visit us in Trento!!
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Save the date: May 8th-12th, 2017
2nd Workshop on Strongly Correlated Fluids of Light and Matter
Cargese, Corse